DISJOINT HYPERCYCLIC POWERS OF WEIGHTED TRANSLATIONS ON GROUPS

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Abstract. Let G be a locally compact group and let $1 \leq p < \infty$. Recently, Chen et al. characterized hypercyclic, supercyclic and chaotic weighted translations on locally compact groups and their homogeneous spaces. There has been an increasing interest in studying the disjoint hypercyclicity acting on different spaces of holomorphic functions. In this note, we will study disjoint hypercyclic and disjoint supercyclic powers of weighted translation operators on the Lebesgue space $L^p(G)$ in terms of the weights. And sufficient and necessary conditions for disjoint hypercyclic and disjoint supercyclic powers of weighted translations generated by aperiodic elements on groups will be given.

Keywords: disjoint hypercyclic, weighted translations, aperiodic element, locally compact group

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1. INTRODUCTION

Let T be a continuous linear self-map on a separable infinite dimensional Banach space X and T^n denote the n-th iterate of T. If there exists a vector $x \in X$ such that the orbit $orb(T, x) = \{T^n x : n = 0, 1, \cdots\}$ is dense in X, then T is called hypercyclic. Such a vector x is said to be hypercyclic for T. Besides, for every pair U, V of nonempty open subsets of X, if there is a non-negative integer m, such that $T^m(U) \cap V \neq \emptyset$, then we call T topologically transitive. It is well known that an operator T is hypercyclic if and only if it is topologically transitive. A stronger condition is the following: the operator T on X is called topologically mixing if for every pair of non-empty open subsets U and V of X there is $m \in \mathbb{N}$ such that $T^n(U)$

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meets V for each $n \geq m$. Hypercyclicity (respectively, supercyclic) has been studied by many authors; we refer to [2, 10, 18] for surveys.

Hypercyclic (respectively, supercyclic) operators $T_1, \cdots, T_N, N \geq 2$, acting on the same space X are said to be disjoint or d-hypercyclic (respectively, d-supercyclic) provided there is some $x \in X$ for which the vector $(x, \cdots, x) \in X^N$ is hypecyclic (respectively, supercyclic) for the direct sum operator $\oplus_{i=1}^N T_i$ acting on the product space X^N, endowed with the product topology. Besides, we say that operators $T_1, \cdots, T_N$ in $B(X)$ are d-topologically transitive provided for any non-empty open subsets $V_0, \cdots, V_N$ of X there exists $m \in \mathbb{N}$ such that

$$V_0 \bigcap T_1^{-m}(V_1) \bigcap \cdots \bigcap T_N^{-m}(V_N) \neq \emptyset.$$

If $T_1, \cdots, T_N$ satisfy the stronger condition that

$$V_0 \bigcap T_1^{-m}(V_1) \bigcap \cdots \bigcap T_N^{-m}(V_N) \neq \emptyset$$

for some m onwards, then $T_1, \cdots, T_N$ are said to be d-mixing. There has been an increasing interest in studying the disjoint hypercyclicity acting on different spaces of holomorphic functions. For example, disjoint hypercyclicity was studied in [1, 3, 4, 16, 17]. Besides, disjoint hypercyclic and supercyclic powers of weighted backward shifts were also characterized in [5, 6, 15].

Recently, hypercyclic, supercyclic and chaotic weighted translations on locally compact groups and their homogeneous spaces were characterized in [8, 9, 7]. And Liang et al. characterized d-hypercyclicity and d-supercyclicity of finite tuples of weighted translations generated by aperiodic elements in [12, 14]. Inspired by their work, we characterize disjoint hypercyclic powers of weighted translations on groups in this paper by developing further the results in [9, 7].

Throughout, let G be a locally compact group with identity e and a right-invariant Haar measure λ. Since a complex Banach space admits a hypercyclic operator if and only if it is separable and infinite-dimensional, the question of hypercyclicity is meaningful for the complex Lebesgue space $L^p(G)$, with respect to λ, only when G is second countable and $1 \leq p < \infty$. A bounded measurable function $w : G \to (0, \infty)$ is called a *weight* on G. Let $a \in G$ and let δ_a be the unit point mass at a. A weighted translation on G is a weighted convolution operator $T_{a,w} : L^p(G) \to L^p(G)$ defined by

$$T_{a,w}(f) := wT_a(f) \quad (f \in L^p(G)),$$

where w is a weight on G and $T_a(f) = f * \delta_a \in L^p(G)$ is the convolution:

$$(f * \delta_a)(x) = \int_G f\left(xy^{-1}\right) d\delta_a(y) = f\left(xa^{-1}\right) \quad (x \in G).$$

An element a in a group G is called a torsion element if it is of finite order. In a locally compact group G, an element $a \in G$ is called periodic [11] (or compact [13, 9.9]) if the closed subgroup $G(a)$ generated by a is compact. We call an element in G aperiodic if it is not periodic. For discrete groups, periodic elements and torsion elements are identical; in other words, aperiodic elements are non-torsion elements. However, non-torsion elements in non-discrete groups need not be aperiodic. It has been shown in [9, Lemma 1.1] that a weighted translation operator is not hypercyclic if it is generated by a torsion element. Our goal in this paper is to characterize disjoint hypercyclic powers of weighted translations generated by aperiodic elements on groups.

2. Disjoint hypercyclic powers of weighted translations

It has been shown in [9, Lemma 2.1] that an elements a in a locally compact group G is aperiodic if and only if, for any compact subset $K \subseteq G$, there exists $m \in \mathbb{N}$ such that $K \cap Ka^n = \emptyset$ (equivalently, $K \cap Ka^{-n} = \emptyset$) for $n > m$. In this section, we will make use of the equivalence of dense d-hypercyclicity and d-topological transitivity [6] to obtain the main result. We are now ready to give sufficient and necessary conditions for disjoint hypercyclic powers of weighted translations generated by aperiodic elements on groups.

Theorem 2.1. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \le p < \infty$ and $1 \le r_1 < r_2 < \cdots < r_N$, where $N \ge 2$, $r_i \in \mathbb{N}$, $i = 1, \cdots, N$. For each $1 \le l \le N$, let $w_l : G \to (0, \infty)$ be a weight on G and T_{a,w_l} be a weighted translation on $L^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are densely d-hypercyclic.

(ii) For $1 \le l \le N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K such that $\lambda(K) = \lim_{k\to\infty} \lambda(E_k)$ and for the sequences

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \tilde{\varphi}_{l,n} := \left(\prod_{s=0}^{r_l n-1} w_l * \delta_a^s \right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty} \|\varphi_{l,n_k}|_{E_k}\|_\infty = \lim_{k\to\infty} \|\tilde{\varphi}_{l,n_k}|_{E_k}\|_\infty = 0, \tag{2.1}$$

and, if $1 \le s < l < N$,

$$\lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod_{t=0}^{r_l n_k-1} w_l * \delta_a^t} |_{E_k} \right\|_\infty = \lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod_{t=0}^{r_s n_k-1} w_s * \delta_a^t} |_{E_k} \right\|_\infty = 0 \tag{2.2}$$

Proof. (ii) $\Rightarrow$ (i). By Proposition 2.3 in [6], we show that $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are d-topologically transitive. Let $V_0, \cdots, V_N$ be non-empty open subsets of $L^p(G)$. Since the space $C_c(G)$ of continuous functions on G with compact support is dense in $L^p(G)$, we can pick $f,\ g_1, \cdots, g_N \in C_c(G)$ with $f \in V_0,\ g_1 \in V_1, \cdots, g_N \in V_N$. Let K be the union of the compact supports of $f,\ g_1, \cdots, g_N$ and let $\chi_K \in L^p(G)$ be the characteristic function of K. For $1 \leq l \leq N$ and a compact subset K of G, let (E_k) and (n_k) as in (2.1) and (2.2).

By the aperiodicity of a, there exists $M \in \mathbb{N}$ such that $K \cap Ka^{\pm n} = \emptyset$ for all $n > M$.

For $1 \leq l \leq N$, we define a self-map S_{a,w_l} on the subspace $L_c^p(G)$ consisting of functions in $L^p(G)$ with compact support by

$$S_{a,w_l}(h) = \frac{h}{w_l} * \delta_{a^{-1}} \quad (h \in L_c^p(G))$$

so that

$$T_{a,w_l}^{r_l n_k} S_{a,w_l}^{r_l n_k}(h) = h \quad (h \in L_c^p(G)).$$

We claim that (2.1) and (2.2) imply the following four equalities:

$$\lim_{k\to\infty} \left\| T_{a,w_l}^{r_l n_k}(f\chi_{E_k}) \right\|_p = 0;$$

$$\lim_{k\to\infty} \left\| S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k}) \right\|_p = 0;$$

$$\lim_{k\to\infty} \left\| T_{a,w_l}^{r_l n_k} S_{a,w_s}^{r_s n_k}(g_s\chi_{E_k}) \right\|_p = 0;$$

$$\lim_{k\to\infty} \left\| T_{a,w_s}^{r_s n_k} S_{a,w_l}^{r_l n_k}(g_l\chi_{E_k}) \right\|_p = 0.$$

We prove the first of the four equalities here; the remaining ones follow similarly. Since $\lim\limits_{k\to\infty} \|\varphi_{l,n_k}|_{E_k}\|_\infty = 0$, given any $\epsilon > 0$, there exists a positive integer $m \in \mathbb{N}$ such that $n_k > M$ and $\varphi_{l,n_k}^p < \frac{\varepsilon}{\|f\|_p^p}$ on E_k if $k > m$. Hence

$$\begin{aligned}
& \left\| T_{a,w_l}^{r_l n_k}(f\chi_{E_k}) \right\|_p^p \\
= & \int_{E_k a^{r_l n_k}} \left| w_l(x) w_l\left(xa^{-1}\right) \cdots w_l\left(xa^{-(r_l n_k - 1)}\right) \right|^p \left| f\left(xa^{-r_l n_k}\right) \right|^p d\lambda(x) \\
= & \int_{E_k} \left| w_l\left(xa^{r_l n_k}\right) w_l\left(xa^{r_l n_k - 1}\right) \cdots w_l(xa) \right|^p |f(x)|^p d\lambda(x) \\
= & \int_{E_k} \left| \varphi_{l,n_k}^p(x) \right| |f(x)|^p d\lambda(x) < \varepsilon, \text{ for } k > m.
\end{aligned}$$

The first equality follows by the arbitrariness of ϵ.

For each $k \in \mathbb{N}$, let

$$v_k = f\chi_{E_k} + \sum_{i=1}^{N} S_{a,w_i}^{r_i n_k}(g_i \chi_{E_k}) \in L^p(G).$$

Then

$$\|v_k - f\|_p^p \leq \|f\|_\infty^p \lambda(K \backslash E_k) + \sum_{i=1}^{N} \left\|S_{a,w_i}^{r_i n_k}(g_i \chi_{E_k})\right\|_p^p$$

and

$$\begin{aligned} \left\|T_{a,w_l}^{r_l n_k} v_k - g_l\right\|_p^p &\leq \left\|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\right\|_p^p + \left\|\sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i \chi_{E_k}) - g_l\right\|_p^p \\ \leq \quad & \left\|T_{a,w_l}^{r_l n_k}(f\chi_{E_k})\right\|_p^p + \|g_l\|_\infty^p \lambda(K \backslash E_k) + \sum_{i \neq l}^{N} \left\|T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}(g_i \chi_{E_k})\right\|_p^p \end{aligned}$$

Hence $\lim_{k\to\infty} v_k = f$ and $\lim_{k\to\infty} T_{a,w_l}^{r_l n_k} v_k = g_l$, which imply

$$V_0 \cap T_{a,w_1}^{-r_1 n_k}(V_1) \cap \cdots \cap T_{a,w_N}^{-r_N n_k}(V_N) \neq \emptyset, \text{ for some } k.$$

(i) $\Rightarrow$ (ii). Let $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ be densely d-hypercyclic. Let $K \subseteq G$ be a compact set with $\lambda(K) > 0$. Let $\varepsilon > 0$. By the aperiodicity of a, there exists $M \in \mathbb{N}$ such that $K \cap Ka^{\pm n} = \emptyset$ for all $n > M$. Let $\chi_K \in L^p(G)$ be the characteristic function of K. Choose $0 < \delta < \frac{\varepsilon}{1+\varepsilon}$. By assumption, there exists a d-hypercyclic vector $f \in L^p(G)$ and some $m > M$ such that for $1 \leq l \leq N$,

$$\|f - \chi_K\|_p < \delta^2 \text{ and } \left\|T_{a,w_l}^{r_l m} f - \chi_K\right\|_p < \delta^2. \tag{2.3}$$

Let $A_\delta = \{x \in K : |f(x) - 1| \geq \delta\}$. Then we have

$$|f(x)| > 1 - \delta \qquad (x \in K \setminus A_\delta) \tag{2.4}$$

and $\lambda(A_\delta) < \delta^p$, since

$$\begin{aligned} \delta^{2p} > \|f - \chi_K\|_p^p &= \int_G |f(x) - \chi_K(x)|^p \, d\lambda(x) \\ &\geq \int_K |f(x) - 1|^p \, d\lambda(x) \\ &\geq \int_{A_\delta} |f(x) - 1|^p \, d\lambda(x) \geq \delta^p \lambda(A_\delta). \end{aligned}$$

Similarly, let $B_\delta = \{x \in G \setminus K : |f(x)| \geq \delta\}$, then we have

$$|f(x)| < \delta \text{ for } x \in (G \setminus K) \setminus B_\delta \tag{2.5}$$

and $\lambda(B_\delta) < \delta^p$.

Let $C_{l,m,\delta} = \{x \in K : \left|\tilde{\varphi}_{l,m}(x)^{-1} f(xa^{-r_l m}) - 1\right| \geq \delta\}$. Then we have

$$\tilde{\varphi}_{l,m}(x)^{-1} \left|f\left(xa^{-r_l m}\right)\right| > 1 - \delta \quad (x \in K \setminus C_{l,m,\delta}) \tag{2.6}$$

and $\lambda(C_{l,m,\delta}) < \delta^p$. In fact,

$$\begin{aligned}
\delta^{2p} &> \left\|T_{a,w_l}^{r_l m} f - \chi_K\right\|_p^p = \int_G \left|T_{a,w_l}^{r_l m} f(x) - \chi_K(x)\right|^p d\lambda(x) \\
&\geq \int_{C_{l,m,\delta}} \left|w_l(x) w_l\left(xa^{-1}\right) \cdots w_l\left(xa^{-(r_l m-1)}\right) f\left(xa^{-r_l m}\right) - 1\right|^p d\lambda(x) \\
&= \int_{C_{l,m,\delta}} \left|\tilde{\varphi}_{l,m}(x)^{-1} f\left(xa^{-r_l m}\right) - 1\right|^p d\lambda(x) \\
&\geq \delta^p \lambda(C_{l,m,\delta}).
\end{aligned}$$

Let $D_{l,m,\delta} = \{x \in K : |\varphi_{l,m}(x) f(x)| \geq \delta\}$. Then we have

$$\varphi_{l,m}(x) |f(x)| < \delta \quad (x \in K \setminus D_{l,m,\delta}) \tag{2.7}$$

and $\lambda(D_{l,m,\delta}) < \delta^p$. In fact, since $K \cap Ka^{r_l m} = \emptyset$, we deduce

$$\begin{aligned}
\delta^{2p} &> \int_G \left|w_l(x) w_l\left(xa^{-1}\right) \cdots w_l\left(xa^{-(r_l m-1)}\right) f\left(xa^{-r_l m}\right) - \chi_K(x)\right|^p d\lambda(x) \\
&= \int_G \left|w_l(xa^{r_l m}) w_l\left(xa^{r_l m-1}\right) \cdots w_l(xa) f(x) - \chi_K(xa^{r_l m})\right|^p d\lambda(x) \\
&\geq \int_{D_{l,m,\delta}} \left|w_l(xa^{r_l m}) w_l\left(xa^{r_l m-1}\right) \cdots w_l(xa) f(x)\right|^p d\lambda(x) \\
&= \int_{D_{l,m,\delta}} |\varphi_{l,m}(x) f(x)|^p d\lambda(x) \\
&\geq \delta^p \lambda(D_{l,m,\delta}).
\end{aligned}$$

Let $F_{l,m,\delta} = \{x \in G \setminus K : \left|\tilde{\varphi}_{l,m}(x)^{-1} f(xa^{-r_l m})\right| \geq \delta\}$. Then we have

$$\left|\tilde{\varphi}_{l,m}(x)^{-1} f\left(xa^{-r_l m}\right)\right| < \delta \text{ for } x \in (G \setminus K) \setminus F_{l,m,\delta} \tag{2.8}$$

and $\lambda\left(F_{l,m,\delta}\right)<\delta^{p}$, since

$$\begin{aligned}
\delta^{2p} &> \int_{G\setminus K}\left|w_l(x)\,w_l\left(xa^{-1}\right)\cdots w_l\left(xa^{-(r_lm-1)}\right)f\left(xa^{-r_lm}\right)\right|^{p}d\lambda(x)\\
&\geq \int_{F_{l,m,\delta}}\left|w_l(x)\,w_l\left(xa^{-1}\right)\cdots w_l\left(xa^{-(r_lm-1)}\right)f\left(xa^{-r_lm}\right)\right|^{p}d\lambda(x)\\
&= \int_{F_{l,m,\delta}}\left|\tilde{\varphi}_{l,m}(x)^{-1}f\left(xa^{-r_lm}\right)\right|^{p}d\lambda(x)\\
&\geq \delta^{p}\lambda\left(F_{l,m,\delta}\right).
\end{aligned}$$

Now, (2.5), (2.6) and the fact $K\cap Ka^{-r_lm}=\emptyset$ imply that

$$\tilde{\varphi}_{l,m}(x)<\frac{\left|f\left(xa^{-r_lm}\right)\right|}{1-\delta}<\frac{\delta}{1-\delta}<\varepsilon \text{ for } x\in K\setminus\left(C_{l,m,\delta}\cup B_\delta a^{r_lm}\right);$$

(2.4) and (2.7) imply that

$$\varphi_{l,m}(x)<\frac{\delta}{|f(x)|}<\frac{\delta}{1-\delta}<\varepsilon \text{ for } x\in K\setminus\left(D_{l,m,\delta}\cup A_\delta\right).$$

By (2.6) and (2.8), for $x\in K\setminus\left(C_{l,m,\delta}\cup a^{(r_l-r_s)m}F_{s,m,\delta}\right)$, we have

$$\begin{aligned}
\frac{w_l(x)\cdots w_l\left(xa^{-(r_lm-1)}\right)}{w_s\left(xa^{1-r_lm}\right)\cdots w_s\left(xa^{r_sm-r_lm}\right)} &= \frac{w_l(x)\cdots w_l\left(xa^{-(r_lm-1)}\right)\left|f\left(xa^{-r_lm}\right)\right|}{w_s\left(xa^{1-r_lm}\right)\cdots w_s\left(xa^{r_sm-r_lm}\right)\left|f\left(xa^{-r_lm}\right)\right|}\\
&= \frac{\tilde{\varphi}_{l,m}(x)^{-1}\left|f\left(xa^{-r_lm}\right)\right|}{\tilde{\varphi}_{s,m}\left(xa^{(r_s-r_l)m}\right)^{-1}\left|f\left(xa^{(r_s-r_l)m}a^{-r_sm}\right)\right|}\\
&> \frac{1-\delta}{\delta}>\frac{1}{\varepsilon},\ \text{ if } 1\leq s<l\leq N.
\end{aligned}$$

Similarly, for $x\in K\setminus\left(C_{s,m,\delta}\cup a^{(r_s-r_l)m}F_{l,m,\delta}\right)$, we have

$$\frac{w_s(x)\cdots w_s\left(xa^{1-r_sm}\right)}{w_l\left(xa^{1-r_sm}\right)\cdots w_l\left(xa^{r_lm-r_sm}\right)}>\frac{1}{\varepsilon},\ \text{ if } 1\leq s<l\leq N.$$

Let

$$\begin{aligned}
\tilde{B}_{m,\delta} &= B_\delta a^{r_1m}\cup\cdots\cup B_\delta a^{r_Nm},\\
\tilde{C}_{m,\delta} &= C_{1,m,\delta}\cup\cdots\cup C_{N,m,\delta},\\
\tilde{D}_{m.\delta} &= D_{1,m,\delta}\cup\cdots\cup D_{N,m,\delta},\\
\tilde{F}_{m.\delta} &= \bigcup_{1\leq s<l\leq N}a^{(r_l-r_s)m}F_{s,m,\delta},\\
\tilde{G}_{m.\delta} &= \bigcup_{1\leq s<l\leq N}a^{(r_s-r_l)m}F_{l,m,\delta}.
\end{aligned}$$

Now, let $H_{m.\delta} = A_\delta \cup \tilde{B}_{m,\delta} \cup \tilde{C}_{m,\delta} \cup \tilde{D}_{m.\delta} \cup \tilde{F}_{m.\delta} \cup \tilde{G}_{m.\delta},\ E_{m,\delta} = K \backslash H_(m,\delta)$. Then $\lambda\left(H_{m\delta}\right) < (1+N)^2\delta^p < (1+N)^2\epsilon^p$ and

$$\left\|\varphi_{l,m}|_{E_{m,\delta}}\right\|_\infty < \varepsilon,\ \ \left\|\tilde{\varphi}_{l,m}|_{E_{m,\delta}}\right\|_\infty < \varepsilon, \tag{2.9}$$

$$\left\| \frac{\prod\limits_{t=1}^{r_s m} w_s * \delta_{a^{-1}}^{t-r_l m}}{\prod\limits_{t=0}^{r_l m-1} w_l * \delta_a^t} |_{E_{m,\delta}} \right\|_\infty < \varepsilon \ ,\ \left\| \frac{\prod\limits_{t=1}^{r_l m} w_l * \delta_{a^{-1}}^{t-r_s m}}{\prod\limits_{t=0}^{r_s m-1} w_s * \delta_a^t} |_{E_{m,\delta}} \right\|_\infty < \varepsilon. \tag{2.10}$$

For $k = 1, 2, \cdots$, take $\epsilon = \frac{1}{k}$ in the above arguments and denote m by n_k, $E_{m,\delta}$ by E_k, then we get a sequence $\{n_k\}$ of positive integers and a sequence $\{E_k\}$ of subsets in K such that $\lambda\left(K\right) = \lim\limits_{k\to\infty} \lambda\left(E_k\right)$ and (2.1) and (2.2) hold. □

By Theorem 2.7 in [6], condition (ii) of Theorem 2.1 also implies that the operators $T_1, \cdots, T_N$ satisfy the d-Hypercyclicity Criterion. Indeed, for each $r \in \mathbb{N}$, one considers non-empty open sets

$$V_{0,j}, V_{1,j}, \cdots, V_{N,j}\ (j = 1, \cdots, r)$$

in $L^p(G)$ and pick $f_{0,j},\ g_{1,j}, \cdots, g_{N,j} \in C_c(G)$ with $f_{0,j} \in V_{0,j},\ g_{1,j} \in V_{1,j}, \cdots, g_{N,j} \in V_{N,j}$, then the same arguments in the proof of (ii) $\Rightarrow$ (i) in Theorem 2.1 can be applied to these functions to obtain r sequences $(v_{1,k}), \cdots, (v_{r,k})$ in $L^p(G)$ satisfying

$$\lim_{k\to\infty} v_{j,k} = f_{0,j} \text{ and } \lim_{k\to\infty} T_{a,w_l}^{r_l n_k} v_{j,k} = g_{l,j} \text{ for } 1 \le l \le N,\ \ 1 \le j \le r,$$

yielding

$$V_{0,j} \cap T_{a,w_1}^{-r_1 n_k}\left(V_{1,j}\right) \cap \cdots \cap T_{a,w_N}^{-r_N n_k}\left(V_{N,j}\right) \neq \emptyset \ \text{ for some } k.$$

Hence we can draw the following result.

Corollary 2.2. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \le p < \infty$ and $1 \le r_1 < r_2 < \cdots < r_N$, where $N \ge 2,\ r_i \in \mathbb{N},\ i = 1, \cdots, N$. For each $1 \le l \le N$, let $w_l : G \to (0, \infty)$ be a weight on G and T_{a,w_l} be a weighted translation on $L^p(G)$. The following conditions are equivalent:*
(i) $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are densely d-hypercyclic.
(ii) $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ satisfy the d-Hypercyclicity Criterion.

Using similar arguments as in the proof of Theorem 2.1, we can also characterize d-topological mixing powers of weighted translations for non-discrete groups.

Corollary 2.3. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \leq p < \infty$ and $1 \leq r_1 < r_2 < \cdots < r_N$, where $N \geq 2,\ r_i \in \mathbb{N},\ i = 1, \cdots, N$. For each $1 \leq l \leq N$, let $w_l : G \to (0, \infty)$ be a weight on G and T_{a,w_l} be a weighted translation on $L^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are d-mixing.

(ii) For $1 \leq l \leq N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K such that $\lambda(K) = \lim_{k\to\infty} \lambda(E_k)$ and the sequences

$$\varphi_{l,k} := \prod_{s=1}^{r_l k} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \tilde{\varphi}_{l,k} := \left(\prod_{s=0}^{r_l k - 1} w_l * \delta_a^s \right)^{-1},$$

satisfy

$$\lim_{k\to\infty} \|\varphi_{l,k}|_{E_k}\|_\infty = \lim_{k\to\infty} \|\tilde{\varphi}_{l,k}|_{E_k}\|_\infty = 0, \tag{2.11}$$

and, if $1 \leq s < l < N$,

$$\lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_s k} w_s * \delta_{a^{-1}}^{t - r_l k}}{\prod_{t=0}^{r_l k - 1} w_l * \delta_a^t} |_{E_k} \right\|_\infty = \lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_l k} w_l * \delta_{a^{-1}}^{t - r_s k}}{\prod_{t=0}^{r_s k - 1} w_s * \delta_a^t} |_{E_k} \right\|_\infty = 0 \tag{2.12}$$

If G is discrete, then $E_m = K$ in the proof of Theorem 2.1. Hence we have the following characterization of disjoint hypercyclic powers of weighted translation operators on discrete groups.

Corollary 2.4. *Let G be a discrete group, and let a be a torsion free element in G. Let $1 \leq p < \infty$ and $1 \leq r_1 < r_2 < \cdots < r_N$, where $N \geq 2,\ r_i \in \mathbb{N},\ i = 1, \cdots, N$. For each $1 \leq l \leq N$, let $w_l : G \to (0, \infty)$ be a weight on G and T_{a,w_l} be a weighted translation on $l^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are densely d-hypercyclic.

(ii) For $1 \leq l \leq N$ and each finite subset $K \subseteq G$, for the sequences

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \tilde{\varphi}_{l,n} := \left(\prod_{s=0}^{r_l n - 1} w_l * \delta_a^s \right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty} \|\varphi_{l,n_k}|_K\|_\infty = \lim_{k\to\infty} \|\tilde{\varphi}_{l,n_k}|_K\|_\infty = 0, \tag{2.13}$$

and, if $1 \leq s < l < N$,

$$\text{(2.14)} \quad \lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod_{t=0}^{r_l n_k - 1} w_l * \delta_a^t} |_K \right\|_\infty = \lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod_{t=0}^{r_s n_k - 1} w_s * \delta_a^t} |_K \right\|_\infty = 0$$

Example. Let $G = \mathbb{Z}$, $a = -1$. For each $1 \leq l \leq N$, we consider the weighted translation T_l on $l^2(\mathbb{Z})$, defined by $T_l = T_{-1, w_l * \delta_{-1}}$, where (w_l) is a sequence of positive weights. Then T_l is a bilateral weighted shift on $l^2(\mathbb{Z})$, that is, $T_l e_j = w_{l,j} e_{j-1}$ with $w_{l,j} = w_l(j)$ for each l. Here $(e_j)_{j\in\mathbb{Z}}$ is the canonical basis of $l^2(\mathbb{Z})$. Let $1 \leq r_1 < r_2 < \cdots < r_N$, where $r_i \in \mathbb{N}$, $i = 1, \cdots, N$. Next, by Corollary 2.4, the operators $T_1^{r_1}, \cdots, T_N^{r_N}$ are densely d-hypercyclic if, and only if, given $\varepsilon > 0$ and $q \in \mathbb{N}$, there exists $m \in \mathbb{N}$ so that for $|j| \leq q$, we have

$$\text{(2.15)} \qquad \begin{cases} \left|\prod_{i=j+1}^{j+r_l m} w_l(i)\right| > \frac{1}{\varepsilon} \\ \left|\prod_{i=j-r_l m+1}^{j} w_l(i)\right| < \varepsilon \end{cases}, \ 1 \leq l \leq N$$

and

$$\text{(2.16)} \quad \begin{cases} \left|\prod_{i=j+1}^{j+r_l m} w_l(i)\right| > \frac{1}{\varepsilon} \left|\prod_{i=j+(r_l-r_s)m+1}^{j+r_l m} w_s(i)\right| \\ \left|\prod_{i=j-(r_l-r_s)m+1}^{j+r_s m} w_l(i)\right| < \varepsilon \left|\prod_{i=j+1}^{j+r_s m} w_s(i)\right| \end{cases}, \ 1 \leq s < l \leq N$$

which are the same with [6, Theorem 4.7].

3. Disjoint supercyclic powers of weighted translations

It is well known that a complex Banach space admits a supercyclic operator if it is one dimensional or infinite-dimensional and separable. Chen [7] characterized supercyclic weighted translation operators on the Lebesgue space $L^p(G)$ in terms of the weight. Inspired by the work, in this section, we will give sufficient and necessary conditions for disjoint supercyclic powers of weighted translations generated by aperiodic elements on groups.

Theorem 3.1. *Let G be a locally compact group, and let a be an aperiodic element in G. Let $1 \leq p < \infty$ and $1 \leq r_1 < r_2 < \cdots < r_N$, where $N \geq 2$, $r_i \in \mathbb{N}$, $i = 1, \cdots, N$. For each $1 \leq l \leq N$, let $w_l : G \to (0, \infty)$ be a weight on G and T_{a,w_l} be a weighted translation on $L^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are densely d-supercyclic.

(ii) For $1 \leq l \leq N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K and there exist sequences $(\alpha_{l,n})_n \subseteq \mathbb{C}\backslash\{0\}$ such that $\lambda(K) = \lim_{k\to\infty} \lambda(E_k)$ and for the sequences

$$\varphi_{l,n} := |\alpha_{l,n}| \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \tilde{\varphi}_{l,n} := \left(|\alpha_{l,n}| \prod_{s=0}^{r_l n-1} w_l * \delta_a^s \right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty} \|\varphi_{l,n_k}|_{E_k}\|_\infty = \lim_{k\to\infty} \|\tilde{\varphi}_{l,n_k}|_{E_k}\|_\infty = 0, \tag{3.1}$$

and, if $1 \leq s < l < N$,

$$\lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod_{t=0}^{r_l n_k-1} w_l * \delta_a^t} |_{E_k} \right\|_\infty = \lim_{k\to\infty} \left\| \frac{\prod_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod_{t=0}^{r_s n_k-1} w_s * \delta_a^t} |_{E_k} \right\|_\infty = 0 \tag{3.2}$$

Proof. (i) $\Rightarrow$ (ii). Let $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ be densely d-supercyclic. Let $K \subseteq G$ be a compact set with $\lambda(K) > 0$. Let $\varepsilon > 0$. By aperiodicity of a, there exists $M \in \mathbb{N}$ such that $K \cap Ka^{\pm n} = \emptyset$ for all $n > M$. Let $\chi_K \in L^p(G)$ be the characteristic function of K. Choose $0 < \delta < \frac{\varepsilon}{1+\varepsilon}$. By assumption, there exists a d-supercyclic vector $f \in L^p(G)$ and some $m > M$ and $\alpha \in \mathbb{C}\backslash\{0\}$ such that for $1 \leq l \leq N$,

$$\|f - \chi_K\|_p < \delta^2 \text{ and } \left\|\alpha T_{a,w_l}^{r_l m} f - \chi_K\right\|_p < \delta^2. \tag{3.3}$$

The rest is similar to the proof of (i) $\Rightarrow$ (ii) in Theorem 2.1, so we omit the details.

(ii) $\Rightarrow$ (i). A simple Baire Category argument and Birkhoff Transitivity Theorem show that $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are densely d-supercyclic provided for every non-empty open subsets $V_0, \cdots, V_N$ of $L^p(G)$, there exist $m \in \mathbb{N}$ and $\lambda_m \in \mathbb{C}\backslash\{0\}$ such that $\emptyset \neq V_0 \cap \lambda_m^{-1} T_{a,w_1}^{-r_1 m}(V_1) \cap \cdots \cap \lambda_m^{-1} T_{a,w_N}^{-r_N m}(V_N)$.

Let $V_0, \cdots, V_N$ be non-empty open subsets of $L^p(G)$. Since the space $C_c(G)$ of continuous functions on G with compact support is dense in $L^p(G)$, we can pick $f,\ g_1, \cdots, g_N \in C_c(G)$ with $f \in V_0,\ g_1 \in V_1, \cdots, g_N \in V_N$. Let K be the union of the compact supports of $f,\ g_1, \cdots, g_N$ and let $\chi_K \in L^p(G)$ be the characteristic function of K. For $1 \leq l \leq N$, let $E_k \subseteq K$ and there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying conditions (3.1) and (3.2).

By aperiodicity of a, there exists $M \in \mathbb{N}$ such that $K \cap Ka^{\pm n} = \emptyset$ for all $n > M$. Similar to the proof of Theorem (2.1), for $1 \leq l \leq N$, we define self-maps S_{a,w_l} on the subspace $L_c^p(G)$ of functions in $L^p(G)$ with compact support by

$$S_{a,w_l}(h) = \frac{h}{w_l} * \delta_{a^{-1}} \quad (h \in L_c^p(G))$$

so that

$$T^{r_l n_k}_{a,w_l} S^{r_l n_k}_{a,w_l}(h) = h \quad (h \in L^p_c(G)).$$

A similar calculation used in Theorem 2.1 will show

$$\lim_{k\to\infty} \left\|\alpha_{l,n_k} T^{r_l n_k}_{a,w_l}(f\chi_{E_k})\right\|_p = 0;$$

$$\lim_{k\to\infty} \left\|\frac{1}{\alpha_{l,n_k}} S^{r_l n_k}_{a,w_l}(g_l\chi_{E_k})\right\|_p = 0;$$

$$\lim_{k\to\infty} \left\|T^{r_l n_k}_{a,w_l} S^{r_s n_k}_{a,w_s}(g_s\chi_{E_k})\right\|_p = 0;$$
$$\lim_{k\to\infty} \left\|T^{r_s n_k}_{a,w_s} S^{r_l n_k}_{a,w_l}(g_l\chi_{E_k})\right\|_p = 0.$$

Hence, we have

$$\lim_{k\to\infty} \left\|T^{r_l n_k}_{a,w_l}(f\chi_{E_k})\right\|_p \left\|S^{r_l n_k}_{a,w_l}(g_l\chi_{E_k})\right\|_p = 0,$$

and

$$\lim_{k\to\infty} \left\|\sum_{i=1}^{N} T^{r_l n_k}_{a,w_l} S^{r_i n_k}_{a,w_i}(g_i\chi_{E_k}) - g_l\chi_{E_k}\right\|_p = 0.$$

By passing to a subsequence if necessary, we may assume that for $1 \leq l \leq N$,

$$\left\|T^{r_l n_k}_{a,w_l}(f\chi_{E_k})\right\|_p \left\|S^{r_l n_k}_{a,w_l}(g_l\chi_{E_k})\right\|_p < \frac{1}{4k^2} \tag{3.4}$$

and

$$\left\|\sum_{i=1}^{N} T^{r_l n_k}_{a,w_l} S^{r_i n_k}_{a,w_i}(g_i\chi_{E_k}) - g_l\chi_{E_k}\right\|_p < \frac{1}{2k} \tag{3.5}$$

Now, let

$$v_k = f\chi_{E_k} + \frac{1}{\alpha_{n_k}} \sum_{i=1}^{N} S^{r_i n_k}_{a,w_i}(g_i\chi_{E_k}) \in L^p(G),$$

where $\alpha_{n_k} := 2k \left\|\sum_{i=1}^{N} S^{r_i n_k}_{a,w_i}(g_i\chi_{E_k})\right\|_p$. Then for $1 \leq l \leq N$,

$$\|v_k - f\|_p \leq \|f\|_\infty \lambda(K \backslash E_k)^{\frac{1}{p}} + \frac{1}{2k},$$

and

$$\left\|\alpha_{n_k} T_{a,w_l}^{r_l n_k} v_k - g_l\right\|_p \leq \left\|\alpha_{n_k} T_{a,w_l}^{r_l n_k}\left(f\chi_{E_k}\right)\right\|_p + \left\|\sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}\left(g_i\chi_{E_k}\right) - g_l\right\|_p$$

$$\leq \left\|\alpha_{n_k} T_{a,w_l}^{r_l n_k}\left(f\chi_{E_k}\right)\right\|_p + \|g_l\|_\infty \lambda\left(K\backslash E_k\right)^{\frac{1}{p}} + \left\|\sum_{i=1}^{N} T_{a,w_l}^{r_l n_k} S_{a,w_i}^{r_i n_k}\left(g_i\chi_{E_k}\right) - g_l\chi_{E_k}\right\|_p$$

$$\leq \frac{1}{2k} + \|g_l\|_\infty \lambda\left(K\backslash E_k\right)^{\frac{1}{p}} + \frac{1}{2k}$$

Hence, $\lim_{k\to\infty} v_k = f$ and $\lim_{k\to\infty} \alpha_{n_k} T_{a,w_l}^{r_l n_k} v_k = g_l$, which imply

$$V_0 \cap \alpha_{n_k}^{-1} T_{a,w_1}^{-r_1 n_k}(V_1) \cap \cdots \cap \alpha_{n_k}^{-1} T_{a,w_N}^{-r_N n_k}(V_N) \neq \emptyset, \text{ for some } k.$$

Therefore, $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are densely d-supercyclic. □

Remark 3.2. By Corollary 2.5 in [7], it is easily shown that the condition (ii) in Theorem 3.1 holds if and only if for $1 \leq l \leq N$ and each compact subset $K \subseteq G$ with $\lambda(K) > 0$, there is a sequence of Borel sets (E_k) in K such that $\lambda(K) = \lim_{k\to\infty} \lambda(E_k)$ and for the sequences

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \tilde{\varphi}_{l,n} := \left(\prod_{s=0}^{r_l n - 1} w_l * \delta_a^s\right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ satisfying

$$\lim_{k\to\infty} \left\|\varphi_{l,n_k} . \tilde{\varphi}_{l,n_k}|_{E_k}\right\|_\infty = 0 \tag{3.6}$$

and (3.2) holds.

If G is discrete, by the proof of Theorem 3.1, we have the following characterization of disjoint supercyclic powers of weighted translation operators on discrete groups.

Corollary 3.3. *Let G be a discrete group, and let a be a torsion free element in G. Let $1 \leq p < \infty$ and $1 \leq r_1 < r_2 < \cdots < r_N$, where $N \geq 2$, $r_i \in \mathbb{N}$, $i = 1, \cdots, N$. For each $1 \leq l \leq N$, let $w_l : G \to (0, \infty)$ be a weight on G and T_{a,w_l} be a weighted translation on $l^p(G)$. The following conditions are equivalent:*

(i) $T_{a,w_1}^{r_1}, \cdots, T_{a,w_N}^{r_N}$ are densely d-supercyclic.

(ii) For $1 \leq l \leq N$ and each finite subset $K \subseteq G$, for the sequences

$$\varphi_{l,n} := \prod_{s=1}^{r_l n} w_l * \delta_{a^{-1}}^s \quad \text{and} \quad \tilde{\varphi}_{l,n} := \left(\prod_{s=0}^{r_l n - 1} w_l * \delta_a^s\right)^{-1}$$

there exists an increasing subsequence $(n_k) \subseteq \mathbb{N}$ *satisfying*

$$\lim_{k\to\infty} \|\varphi_{l,n_k} \cdot \tilde{\varphi}_{l,n_k}|_K\|_\infty = 0 \tag{3.7}$$

and, if $1 \le s < l < N$,

$$\lim_{k\to\infty} \left\| \frac{\prod\limits_{t=1}^{r_s n_k} w_s * \delta_{a^{-1}}^{t-r_l n_k}}{\prod\limits_{t=0}^{r_l n_k -1} w_l * \delta_a^t} |_K \right\|_\infty = \lim_{k\to\infty} \left\| \frac{\prod\limits_{t=1}^{r_l n_k} w_l * \delta_{a^{-1}}^{t-r_s n_k}}{\prod\limits_{t=0}^{r_s n_k -1} w_s * \delta_a^t} |_K \right\|_\infty = 0 \tag{3.8}$$

Example. Let $G = \mathbb{Z}$, $a = -1$. For each $1 \le l \le N$, we consider weighted translation T_l on $l^2(\mathbb{Z})$, defined by $T_l = T_{-1, w_l * \delta_{-1}}$, where (w_l) is a positive weight. Then T_l is a bilateral weighted shift on $l^2(\mathbb{Z})$, that is, $T_l e_j = w_{l,j} e_{j-1}$ with $w_{l,j} = w_l(j)$ for each l. Here $(e_j)_{j\in\mathbb{Z}}$ is the canonical basis of $l^2(\mathbb{Z})$. Let $1 \le r_1 < r_2 < \cdots < r_N$, where $r_i \in \mathbb{N}$, $i = 1, \cdots, N$. Next, by Corollary 3.3, the operators $T_1^{r_1}, \cdots, T_N^{r_N}$ are densely d-supercyclic if, and only if, given $\varepsilon > 0$ and $q \in \mathbb{N}$, there exists $m \in \mathbb{N}$ so that for $|j| \le q$, $|k| \le q$ and $1 \le s, l \le N$ we have

$$\left| \prod_{i=j-r_l m+1}^{j} w_l(i) \right| < \varepsilon \left| \prod_{i=k+1}^{k+r_s m} w_s(i) \right| \quad (1 \le l, s \le N), \tag{3.9}$$

and

$$\left\{ \begin{array}{l} \left| \prod_{i=j+1}^{j+r_l m} w_l(i) \right| > \frac{1}{\varepsilon} \left| \prod_{i=j+(r_l-r_s)m+1}^{j+r_l m} w_s(i) \right| \\ \left| \prod_{i=j-(r_l-r_s)m+1}^{j+r_s m} w_l(i) \right| < \varepsilon \left| \prod_{i=j+1}^{j+r_s m} w_s(i) \right| \end{array} \right. , 1 \le s < l \le N \tag{3.10}$$

which are the same with [15, Theorem 4.2.1].

References

[1] *L. Bernal-González*: Disjoint hypercyclic operators. Studia Math. **182 (2)** (2007) 113-131.

[2] *F. Bayart, E. Matheron*: Dynamics of Linear Operators, Camberidge University Press, 2009.

[3] *J. Bès, Ö. Martin, A. Peris*: Disjoint hypercyclic linear fractional composition operators, J. Math. Anal. Appl. **381** (2011) 843-856.

[4] *J. Bès, Ö. Martin, A. Peris, S. Shkarin*: Disjoint mixing operators. J. Funct. Anal. **263** (2012) 1283-1322.

[5] *J. Bès, Ö. Martin, R. Sanders*: Weighted shifts and disjoint hypercyclicity, J. Operator Theory **72(2)** (2014) 15-40.

[6] *J. Bès, A. Peris*: Disjointness in hypercyclicity, J. Math. Anal. Appl. **336** (2007) 297-315.

[7] *C.C. Chen*: Supercyclic and cesàro hypercyclic weighted translations on groups, Taiwanese J. Math. **16 (5)** (2012) 1815-1827.

[8] *C.C. Chen, C.H. Chu*: Hypercyclicity of weighted convolution operators on homogeneous spaces, Proc. Amer. Math. Soc. **137** (2009) 2709-2718.

[9] *C.C. Chen, C.H. Chu*: Hypercyclic weighted translations on groups, Proc. Amer. Math. Soc. **139** (2011) 2839-2846.
[10] *K.G. Grosse-Erdmann, A. Peris Manguillot*: Linear Chaos, Springer, New York, 2011.
[11] *S. Grosser, M. Moskowitz*: On central topological groups, Trans. Amer. Math. Soc. **127** (1967) 317-340.
[12] *S.A. Han, Y. X. Liang*: Disjoint hypercyclic weighted translations generated by aperiodic elements, Collect. Math. **67 (3)** (2016) 347-356.
[13] *E. Hewitt, K.A. Ross*: Abstract harmonic analysis, Springer-Verlag, Heidelberg, 1979.
[14] *Y.X. Liang, L. Xia,*: Disjoint supercyclic weighted translations generated by aperiodic elements, Collect. Math. (2016). DOI:10.1007/s13348-016-0164-4.
[15] *Ö. Martin*: Disjoint hypercyclic and supercyclic composition operators, PhD thesis, Bowling Green State University, 2010.
[16] *H.N. Salas*: Dual disjoint hypercyclic operators, J. Math. Anal. Appl. **374** (2011) 106-117.
[17] *S. Shkarin*: A short proof of existence of disjoint hypercyclic operators, J. Math. Anal. Appl. **367** (2010) 713-715.
[18] *L. Zhang, Z.H. Zhou*: Notes about the structure of common supercyclic vectors, J. Math. Anal. Appl. **418** (2014) 336-343.

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